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The Relation between the Velocity and Mass Distributions. The Role of Collisionless Relaxation Processes

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We consider steady-state mass distributions (mass functions) attained at the nonlinear stage of fragmentation as a result of fragment coalescence. The influence of the fragment velocity distribution on the mass function is discussed. The kinetic equations governing the fragments "quasiparticles" are solved using the group symmetry properties of the collision integral. We have calculated power indices and locations of the breaking points for the mss spectrum (luminosity function) associated with the transition from the collisionless Lynden-Bell type to the Maxwellian distribution, as well as with the predominance of either purely geometric or Newtonian collision cross sections. The power indices found are in a reasonable agreement with the values observed for star or galaxy clusters.

KEY WORDS: Kinetic equations; collision integral; symmetry transformations; power-law spectra; coalescence; galaxay clusters; star clusters.

1. INTRODUCTION

The mass spectrum determining the luminosity function of galaxy or star clusters currently is believed to be formed at the nonlinear stage of fragmentation, with the dominant process being the coalescence of fragments in collisions, provided the fragment masses are not too small (see the reviews in Refs. 1 and 2). This renders reasonable attempts to determine the said portion of the spectrum by solving the coagulation equation (see, e.g., Refs. 3–7). That equation can be derived by averaging over velocities the kinetic equations which govern the behavior of fragments treated as particles possessing momentum and mass, i.e., values conserved during collisions. The

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velocity distribution influences the mass distribution (in particular, owing to the fact that the collision cross section depends on relative velocity). Large-scale fluctuations are subject to a rapid collisionless relaxation via self-consistent gravitation fields.[8] As a result a steady-state velocity distribution is attained which has, in the simplest case, the same form as the equilibrium distribution although with a "temperature" proportional to masses m of the constituent particles. Thereby the mass is not involved in the distribution function, which is characteristic of motion in a purely gravitational field, viz.,

$$\chi_L\left(\frac{\mathbf{p}}{m}\right) \sim \exp\left(-\frac{v^2}{v_L^2}\right) \tag{1}$$

The substantially slower collisional relaxation results in the Maxwellian distribution

$$\chi_M\left(\frac{\mathbf{p}}{\sqrt{m}}\right) \sim \exp\left(-\frac{p^2}{2mT}\right) \tag{2}$$

with T expressed in energy units.

Since the equilibrium distribution Eqs. (1) and (2) depend on the ratio of momentum to some power of mass, it is possible to obtain self-similar solutions of the kinetic equation. The same is true for nonequilibrium stationary (or quasistationary) distributions of similar form.

2. KINETIC EQUATION IN THE SPACE OF MOMENTA AND MASSES

The solution of the kinetic equation for the momentum and mass distribution $f(m, \mathbf{p})$, which will be discussed below, makes use of group (symmetry) properties of the equation.[9–13] For binary collisions resulting in fragment coalescence, the equation can be represented as

$$\begin{aligned}\dot{f}(q) = \int dq_1\, dq_2 [&W(q \mid q_1 q_2) f(q_1) f(q_2) \\ &- W(q_1 \mid q q_2) f(q) f(q_2) - W(q_2 \mid q q_1) f(q) f(q_1)]\end{aligned} \tag{3}$$

where $q \equiv (m, \mathbf{p})$. We do not allow for processes in which mass is not conserved (such as accretion, mass losses during collisions, etc.); therefore the probability W is proportional to the δ functions responsible for mass and momentum conservation, viz.,

$$W(q \mid q_1 q_2) = U(q \mid q_1 q_2)\, \delta(m - m_1 - m_2)\, \delta(\mathbf{p} - \mathbf{p}1 - \mathbf{p}_2) \tag{4}$$

The amplitude U_q is proportional to the collision cross section and the relative velocity v. For spherical fragments of radii r_1 and r_2 it is.[14]

$$U(q \mid q_1 q_2) = \pi(r_1 + r_2)^2 \left[1 + 2G \frac{m_1 + m_2}{v^2(r_1 + r_2)} \right] v \tag{5}$$

With a fixed density of fragments $r \sim m^{1/3}$, however, along with this case, important though it is, we shall consider the generalization $r \sim m^{\beta}$.

In the limiting cases of impact $(Gm/rv^2 \ll 1)$ or purely gravitational $(Gm/rv^2 \gg 1)$ interaction, and with the density being a power-law function of mass, U_q is a homogeneous function of both masses and momenta, hence it can be represented symbolically as $U_q \sim m^{\delta} |\mathbf{p}|^{\zeta}$, or explicitly as

$$\begin{aligned} &U(\lambda m, \mu\mathbf{p} \mid \lambda m_1, \mu\mathbf{p}_1; \lambda m_2, \mu\mathbf{p}_2) \\ &\quad = \lambda^{\delta} \mu^{\zeta} U(m, \mathbf{p} \mid m_1, \mathbf{p}_1; m_2, \mathbf{p}_2) \end{aligned} \tag{6}$$

with

$$\delta = \begin{cases} 2\beta - 1 \\ \beta + 2 \end{cases} \quad \text{and} \quad \zeta = \begin{cases} +1 \\ -1 \end{cases} \quad \text{for} \quad \frac{Gm}{rv^2} \begin{cases} \ll 1 \\ \gg 1 \end{cases}$$

Before going over to the analysis of the coagulation equation, let us dwell shortly on Eq. (3). As long as the homogeneity condition of Eq. (6) holds, the collision integral is characterized by a nontrivial symmetry[11–13] permitting one to obtain a Kolmogorov-type nonequilibrium steady-state solution. The solution, a power-law function of its arguments, $f(q) \sim m^k |\mathbf{p}|^l$, corresponds to a constant mass flux J (since the mass increases in the coalescence processes, the flux is directed from smaller to larger masses).

To derive the solution, we employ in the second and third terms of the integrand in Eq. (3), respectively, the following transformations[10–12,15] (see Fig. 1):

$$\begin{aligned} &\hat{G}_2: m_1 \to \left(\frac{m}{m_2}\right) m, \quad m_2 \to \left(\frac{m}{m_2}\right) m_1; \quad \mathbf{p}_1 \to \left(\frac{p}{p_2} \hat{g}_2\right)^2 \mathbf{p}_2, \quad \mathbf{p}_2 \to \frac{p}{p_2} \hat{g}_2 \hat{g}_2 \mathbf{p}_1 \\ &\hat{G}_1: m_2 \to \left(\frac{m}{m_1}\right) m, \quad m_1 \to \left(\frac{m}{m_1}\right) m_2; \quad \mathbf{p}_2 \to \left(\frac{p}{p_1} \hat{g}_1\right)^2 \mathbf{p}_1, \quad \mathbf{p}_1 \to \frac{p}{p_1} \hat{g}_1 \mathbf{p}_2 \end{aligned} \tag{7}$$

which have the meaning of extensions and rotations

$$\hat{g}_i \left(\hat{g}_i \frac{\mathbf{p}_i}{p_i} = \frac{\mathbf{p}}{p} \right), \qquad i = 1, 2$$

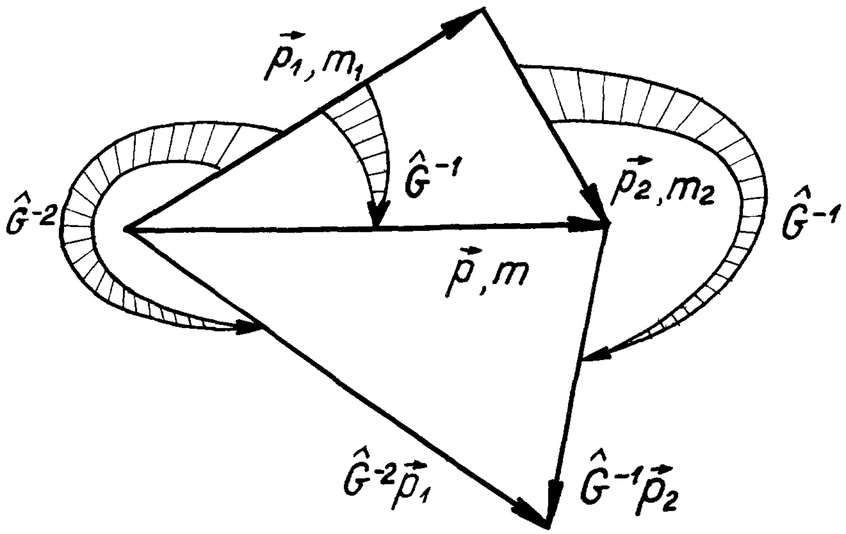

Fig. 1. Symmetry transformations for the collision integral, including extensions and rotations, which do not change fixed values of m and $\mathbf{p}$.

in the m and $\mathbf{p}$ spaces. Taking into account the invariance of the probability function during rotations in the p space, the collision integral can be factorized, i.e.,

$$I_{\text{st}}\{f(q)\} = \int dq_1\, dq_2\, W(q \mid q_1 q_2) \times \left[1 - \left(\frac{m}{m_1}\right)^{\omega} \left(\frac{p}{p_1}\right)^{\eta} - \left(\frac{m}{m_2}\right)^{\omega} \left(\frac{p}{p_2}\right)^{\eta}\right] \tag{8}$$

with $\omega = 2 + \delta + 2k$, $\eta = 6 + \zeta + 2l$.

With $\omega = -1$ and $\eta = 0$, the integral nullifies by virtue of the mass conservation law, yielding the following index values to describe steady-state isotropic distributions: $k = -(\delta + 3)/2$ and $l = -(\zeta + 6)/2$.

Thus, for impact and gravitational interactions, flux-dependent mass and momentum distributions have the form

$$f(m, \mathbf{p}) \sim \begin{cases} m^{-(\beta+1)} p^{-7/2} \\ m^{-(\beta+5)} p^{-5/2}, \end{cases} \qquad \frac{Gm}{rv^2} \begin{cases} \ll 1 \\ \gg 1 \end{cases} \tag{9}$$

3. COAGULATION EQUATION FOR THE MASS DISTRIBUTION FUNCTION

The distributions of Eq. (9) can be formed if both the velocity and mass relaxation times are of the same order of magnitude. Consider now a system where the velocity relaxation is faster than the mass relaxation. Then the distribution can reach an equilibrium with respect to velocities and the coalescence process is controlled by the coagulation equation for the mass distribution function, $f_m = \int d\mathbf{p}\, f(m, \mathbf{p})$:

$$\dot{f}_m = \int dm_1\, dm_2 [W_{m|m_1m_2} f_{m_1} f_{m_2} - W_{m_1|m_2m} f_{m_2} f_m + W_{m_2|mm_1} f_m f_{m_1}] \tag{10}$$

where

$$W_{m|m_1m_2} = U_{m|m_1m_2}\, \delta(m - m_1 - m_2) \tag{11}$$

It can be obtained by averaging Eq. (3) over momenta. Now we represent the distribution function $f(m, \mathbf{p})$ as

$$f(m, \mathbf{p}) = m^{-3\alpha} f_m \chi\left(\frac{\mathbf{p}}{m^\alpha}\right) \tag{12}$$

[$\alpha = 1$ for the Lynden–Bell case of Eq. (1) and $\alpha = 1/2$ for the Maxwellian case of Eq. (2)]. The first factor in Eq. (12) has been introduced for the convenience of normalizing the momentum distribution, viz.,

$$\int d\pi\, \chi(\boldsymbol{\pi}) = 1 \qquad \text{where} \quad \boldsymbol{\pi} = \mathbf{p}/m^\alpha \tag{13}$$

The coagulation probability involved in Eq. (11) is

$$U_{m|m_1m_2} = \int d\boldsymbol{\pi}\, d\boldsymbol{\pi}_1\, d\boldsymbol{\pi}_2\, U(q \mid q_1 q_2)\, \delta(\mathbf{p} - \mathbf{p}_1 - \mathbf{p}_2)\, \chi(\boldsymbol{\pi}_1)\, \chi(\boldsymbol{\pi}_2) \tag{14}$$

The homogeneity of U_q entails a homogeneity of U_m, which is governed not by fragment masses alone but by the velocity distribution of Eq. (12) as well, i.e.,

$$U_{\lambda m|\lambda m_1, \lambda m_2} = \lambda^u U_{m|m1\, m_2}, \qquad u = \delta + \alpha\zeta \tag{15}$$

Here again we restrict ourselves to the steady-state solution corresponding to a constant mass flux $\mathscr{J}$ in the mass space which implies the presence of a source at smaller masses.[3] If Eq. (15) holds, such a solution is a power function $f_m \sim m^s$; it can serve as a sufficiently good intermediate approximation valid up to the nonstationary edge at larger masses and down to accretion-controlled regions at smaller masses [2] (where mass is not conserved in collisions).

Upon tranforming the masses as per Eq. (7) we can factorize the integrand of Eq. (10) to become

$$I_{\text{st}}\{f_m\} = \int dm_1\, dm_2\, W_{m|m_1m_2} f_{m_1} f_{m_2} \left[1 - \left(\frac{m}{m_1}\right)^v - \left(\frac{m}{m_2}\right)^v\right] \qquad (16)$$

with $v = 2 + u + 2S$. For $v = -1$ we obtain the steady-state solution:

$$f_m = Am^s, \qquad s = -\frac{3+u}{2} = -\frac{3+\delta+\alpha\zeta}{2} \qquad (17)$$

Flux-dependent mass distributions for impact and gravitational interactions have the form

$$f_m \sim J^{1/2} \begin{cases} m^{-(\beta+1+\alpha/2)} \\ m^{-(\beta+5-\alpha)/2}, \end{cases} \quad \frac{Gm}{r\bar{v}^2} \begin{cases} \ll 1 \\ \gg 1 \end{cases} \qquad (18)$$

$\bar{v}$ being the characteristic velocity. The relevant proportionality factors can be calculated as in Ref. 15.

As can be seen in going over from the impact to gravitational interaction the power index of the distribution changes by $\Delta S = -(\beta + 2\alpha - 3)/2$. With $\alpha = (3-\beta)/2$ a power-law mass distribution without kinks is possible, as a result of the homogeneity of U_{m/m_1m_2} in the case where the complete probability $U(q \mid q_1 q_2)$ of Eq. (5) satisfies the generalized homogeneity condition

$$U(\lambda m, \lambda^\kappa \mathbf{p} \mid \lambda m_1, \lambda^\kappa \mathbf{p}_1; \lambda m_2, \lambda^\kappa \mathbf{p}_2) = \lambda^\xi U(q \mid q_1 q_2), \qquad \kappa = \frac{3-\beta}{2} \qquad (19)$$

and in the function of Eq. (12) $\alpha = \kappa$.

The power indices for flux-dependent distributions of Eq. (18) for the constant-density case (i.e., $\beta = 1/3$) are summarized in Table 1. They are different from those found previously for zero-flux power-law mass functions(3–7) representing the evolution of initial distributions in the absence of an external source.

[3] Solutions in the presence of a source were considered in the theory of a weak turbulence,(10–13) as well as in problems pertaining to aerosol coagulation though by different methods, e.g. Ref. 16–18. In Kharkov, the efforts in these directions were supported by I. M. Lifschitz.

Table 1

Density		Interaction type	
$\beta = 1/3$		Gravitational $Gm/r\bar{v}^2 \gg 1$	Contact $Gm/r\bar{v}^2 \ll 1$
Relaxation mechanism:	Lynden-Bell, $\alpha = 1$	$S = -13/6$ -2.17	$S = -11/6$ -1.833
	Maxwell, $\alpha = 1/2$	$S = -29/12$ -2.42	$S = -19/12$ -1.58

4. LOCALITY AND STABILITY OF THE STEADY-STATE MASS SPECTRUM

The convergence of the collision integral in Eq. (10) over flux-dependent distributions, which disctates their locality,[10–12,15] depends on the behavior of the coagulation probability (14) with $m_1 \ll m$, m_2, i.e.,

$$U_{m|m_1 m_2} \sim m_1^{u_1} m_2^{u_2} \tag{20}$$

The condition of convergence at $m_1 \to 0$ is

$$u - 2u_1 - 1 < 0 \tag{21}$$

moreover this condition guarantees convergence at larger masses $m_1 \to \infty$. For arbitrary α and β it reduces to the inequality

$$\alpha\zeta - 2(\alpha - 1)\,\theta(1 - \alpha) - \frac{1 + 3\zeta}{2} + \frac{3 + \zeta}{2}\,|\beta| < 0 \tag{22}$$

where $\theta(x)$ is the Heaviside function.

As follows from Eq. (21), the flux-dependent distributions of Eq. (18) formed by the collisional relaxation ($\alpha = 1/2$) will be local if

$$|\beta| = \frac{1}{2}, \qquad \frac{Gm}{r\bar{v}^2} \gg 1, \qquad |\beta| < \frac{1}{4}, \qquad \frac{Gm}{r\bar{v}^2} \ll 1 \tag{23}$$

whereas in the case of a collisionless relaxation ($\alpha = 1$) if

$$|\beta| < \frac{1}{2}, \qquad \frac{Gm}{r\bar{v}^2} \ll 1 \tag{24}$$

Among the constant-density mass spectra represented in Table 1, local are those with power indices equal to $S = -29/12$ and $-11/6$.

Now we shall analyze the stability of local steady-state distributions of Eq. (17) against small perturbations. Let

$$f_m(t) = f_m^{(0)} + \delta f_m(t) \tag{25}$$

where $f_m^{(0)}$ is the steady-state distribution. By linearizing Eq. (10) in $\delta f_m(t)$ and making use of the homogeneity of Eq. (15), we obtain for the Mellin transform of the perturbation

$$\begin{aligned} \phi(z, t) &= \int_0^\infty dm\, m^{z-1} F(m, t) \\ F(m, t) &\equiv \delta f_m(t)/f_m^{(0)} \end{aligned} \tag{26}$$

a linear differential-difference equation

$$\frac{\partial \Phi(z, t)}{\partial \tau} = \Lambda\left(z + \frac{u-1}{2}\right) \Phi\left(z + \frac{u-1}{2}, t\right) \tag{27}$$

$$\Lambda(z) = A^{-1} \int_0^1 dx\, U_{1|x,1-x} x^s (1-x)^s (x^{-z} + (1-x)^{-z})[1-x)^{z+1} - x^{z+1}] \tag{28}$$

With $u = 1$, Zq. (27) becomes a differential equation for which the solution to the problem of evolution of the initial perturbation is

$$\delta f_m(t) = f_m^{(0)} \int_0^\infty \frac{dx}{x} F(mx, 0) \int_{c-i\infty}^{c+i\infty} \frac{dz}{2\pi i} x^z e^{i\Lambda(z)t} \tag{29}$$

Let the initial perturbation be localized, i.e., satisfy the condition

$$\lim_{m \to 0, \infty} \frac{\delta f_m(0)}{f_m^{(0)}} = 0 \tag{30}$$

Then, as follows from Eq. (29) and the analytic properties of $\Lambda(z)$ determined by the asymptotic behavior of $P_{m/m_1 m_2}$ [see Eq. (20)], the contribution to Eq. (29) comes mainly from samll Z's, i.e.,

$$\delta f_m(t) \sim f_m^{(0)} F(me^{-t\Lambda'(0)}, 0), \qquad \Lambda'(0) \equiv \left.\frac{d\Lambda}{dz}\right|_{z=0} > 0 \tag{31}$$

Hence, $\delta f_m \to 0$ as $t \to \infty$, i.e., i.e., the initial distribution is stable. Here we stress that locality is sufficient to make a steady-state flux-dependent distribution stable. Note that the decay of the perturbation with $t \to \infty$ is conditioned by the decrease of the initial perturbation at $m \to 0$ [cf. Eq. (30)], since relaxation tends to shift the perturbation towards larger masses. This gives grounds to expect stability also with $u \neq 1$ when a direct analysis is too complicated.

5. MASS SPECTRA OF GALACTIC AND STELLAR CLUSTERS

To get an insight into the nature of the mass spectrum, let us restrict the consideration by fragments of equal density, comparing the rates of various relaxation processes. The result for a constant density ($\beta = 1/3$) is plotted in Fig. 2, showing, on the mass vs. fragment velocity plane, lines corresponding to equality of any two relaxation times. The broken curves confine the (shaded) region of moderate masses and "intermediate" velocities, where collisionless relaxation dominates over collisions which is typical of the systems of interest to us.[4]

[4] We assume for the collisionless relaxation $\tau_\Lambda \sim (G\bar{\rho})^{-1/2}$. Though this estimate is correct only for sufficiently strong initial fluctuations [12, 19], this seems to be a natural assumption in our case.

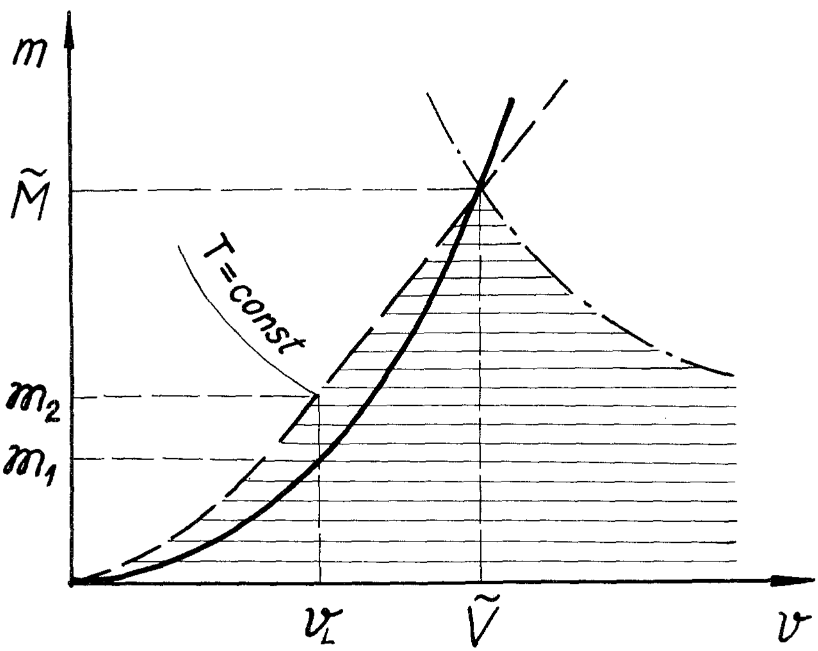

Fig. 2. Competition of relaxation processes. The solid line demarcates regions with essentially gravitational (to the left) and contact (to the right) interactions. The broken line demarcates regions of domination, respectively, of the collisionless (right) and gravitational (left) relaxation. The dot-dashed curve demarcates regions of domination, respectively, of the contact (right) and Lynden-Bell type relaxation.

The broken lines intersect at point $(\tilde{M}, \tilde{V})$ whose position depends on both the mean density $\bar{\rho}$ of the system and the fragment density ρ_0. Specifically, if R is the mean separation between fragments and $\bar{\rho}/\rho_0 \simeq (r/R)^3 \ll 1$ then $\tilde{M} \equiv R^3(\rho_0\bar{\rho})^{1/2}$ and $\tilde{V} \equiv R(G^3\rho_0^2\bar{\rho})^{1/6}$. To the left from the line, Newtonian collisions due to the gravitational interaction dominate (with the characteristic time $\tau_G = (R^3/r^2v)(Gm/r\bar{v}^2)^{-2}$).

These tend to form Maxwellian-like velocity distributions. The same point $(\tilde{M}, \tilde{V})$ also belongs to the solid curve where interaction times of purely contact and Newtonian collisions are equal. In the Lynden-Bell region this curve demarcates the region of "smaller" masses where contact collisions are essential from that of "larger" masses where Newtonian collisions, resulting in coalescence and forming the mss spctrum, are essential. This is why collisions of the latter type, next in rate process after the collisionless relaxation, are important.[5]

For rough estimates we can assume the distributions to be localized near the lines $V = V_L < \tilde{V}$ (for the Lynden-Bell case) and $mv^2/2 = T$ (for the Maxwellian case). Then we find that in the collisionless relaxation domain $(m < \mathfrak{M}_2)$, coalescence is controlled by impact collisions for $m < \mathfrak{M}_1$ or Newtonian collisions for $\mathfrak{M}_1 < m < \mathfrak{M}_2$. In the range of collisional relaxation $(m > \mathfrak{M}_2)$ the major role belongs to gravitational interactions resulting in fragment coalescence.

The power index grows with increasing m as $-S = 11/6 \to 13/6 \to 29/12$ (cf. Fig. 3a). Kinks (break points) of the spectrum correspond to the following mass values:

$$\begin{aligned} \mathfrak{M}_1 &= M(v_L/V)^3 = (G^3\rho_0)^{-1/2} v_L^3 \\ \mathfrak{M}_2 &= \tilde{M}(v_L/\tilde{V})^{3/2} = (G^{-3}\bar{\rho})^{1/4}(Rv_L)^{3/2} \end{aligned} \tag{32}$$

with $M = R^3\rho_0$ and $V = R(G\rho_0)^{1/2}$.

Comparing the calculated power indices with the values observed[2,20] for stars we see that the calculated values $S = -1.8$; -2.4 are in a rather satisfactory agreement with the observations (the average index $S = -2.5$,[2] some portion of the observed spectrum have $S = -1.8$).

The transient effects neglected in the analysis should result in a cutoff (sharper slopes) of the spectrum at larger masses while violation of the mass conservation law should manifest itself at smaller masses.

At point m_2, the power index changes by $\Delta S = 1/4$, the steeper dependence (Newtonian interaction) corresponding to the transition from

[5] Strictly speaking, the Lynden-Bell relaxation forms the distribution of sufficiently large-scale fluctuations whereas the true fragment distribution may differ from the one employed. The latter, however, yields the required distribution after being averaged over fluctuation scales. The mass spectrum is assumed to be accordingly smoothed.

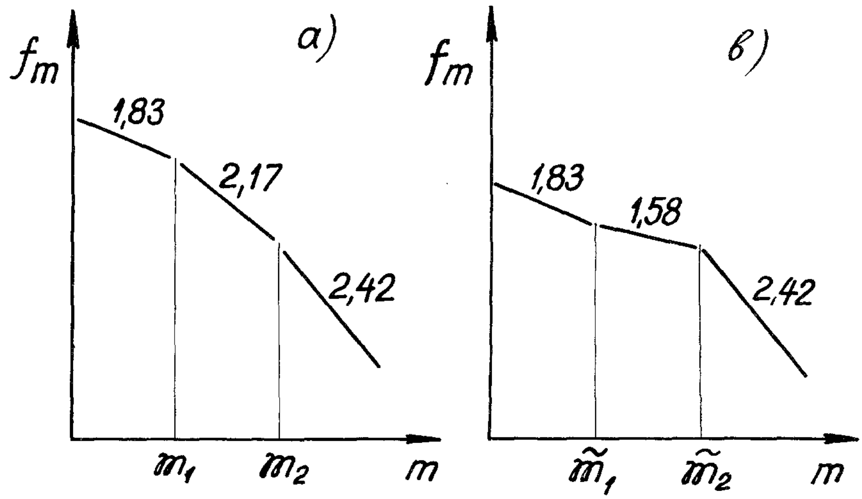

Fig. 3. The mass spectrum for (a) $v_L < \tilde{V}$ and (b) $v_L > \tilde{V}$. The figures are power index values ($-S$).

collisionless to collision-controlled distributions. In the case of contact interactions the transition from the Lynden-Bell to the Maxwellian distribution would result in a flattening of the spectrum, with a correspondent change of the power index by $\Delta S = -1/4$.

The topography of regions corresponding to different relaxation mechanisms and positions of the kinks can be influenced by the law of density variation, i.e., the value of β.[6]

Considering galaxy clusters, $v_1 > \tilde{V}$ cannot be excluded. The situation is aggravated by the fact that collisions are likely to form nonequilibrium distributions (since the conditions are nonstationary and an equivalent source s present). Nonetheless, let us assume that a Maxwellian distribution is established in the case. Then, circumventing the node $(\tilde{M}, \tilde{V})$ from the side of larger velocities and msses, we obtain the following sequence of indices with the increase of mass (Fig. 3b): $-S = 11/6 \rightarrow 19/12 \rightarrow 29/12$, the positions of kinks being

$$\tilde{\mathfrak{M}}_1 = \tilde{M}(v_L/\tilde{V})^{-3/2} = (G^2 \rho_0^4 \bar{\rho}^3)^{1/4} (R^{-3} v_L)^{-3/2} \tag{33}$$

$$\tilde{\mathfrak{M}}_2 = \tilde{M}(v_L/\tilde{V})^{3/10} = (G^{-3} \rho_0^8 \bar{\rho}^9)^{1/20} (R^9 v_L)^{3/10} \tag{34}$$

The plot of Fig. 3b resembles the luminisity function for Comal Berenices[(21)] while Abell's kink[(22)] rather corresponds to Fig. 3b.

[6] It can be seen that the kinks associated with changes in the distribution form for the same coalescence mechanism are universal (independent of β).

It should be noted that in the mass spectra calculated, the property of locality is revealed by those asymptotics which correspond either to the smallest

$$\left(m < \begin{cases}\mathfrak{M}_1 \\ \tilde{\mathfrak{M}}_1\end{cases}, \qquad v_L \begin{cases} < \\ > \end{cases} \tilde{V}\right)$$

or ultimately large

$$\left(m > \begin{cases}\mathfrak{M}_2 \\ \tilde{\mathfrak{M}}_2\end{cases}, \qquad v_L \begin{cases} < \\ < \end{cases} \tilde{V}\right)$$

masses. The nonlocality of power law asymptotics in the intermediate mass range seems to be of little importance, since the limiting cases $m \to 0, \infty$ yield local distributions.

Rough estimates of relaxation times confirm the reality of the mechanisms considered.

Obviously enough the analysis presented cannot be claimed as a complete description of the luminosity function (mass function) (cf. Ref. 22 where mechanisms neglected by these authors are discussed). Yet the results obtained which correspond qualitatively to typical initial mass functions (for velocities of the colliding fragments insufficient for crushing) suggest that coalescence might be a critical factor in shaping the initial mass distribution, at least over some range of masses.

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